Frequency Weightings for Sound and Vibration Perceived by Humans using Multi-Instrument

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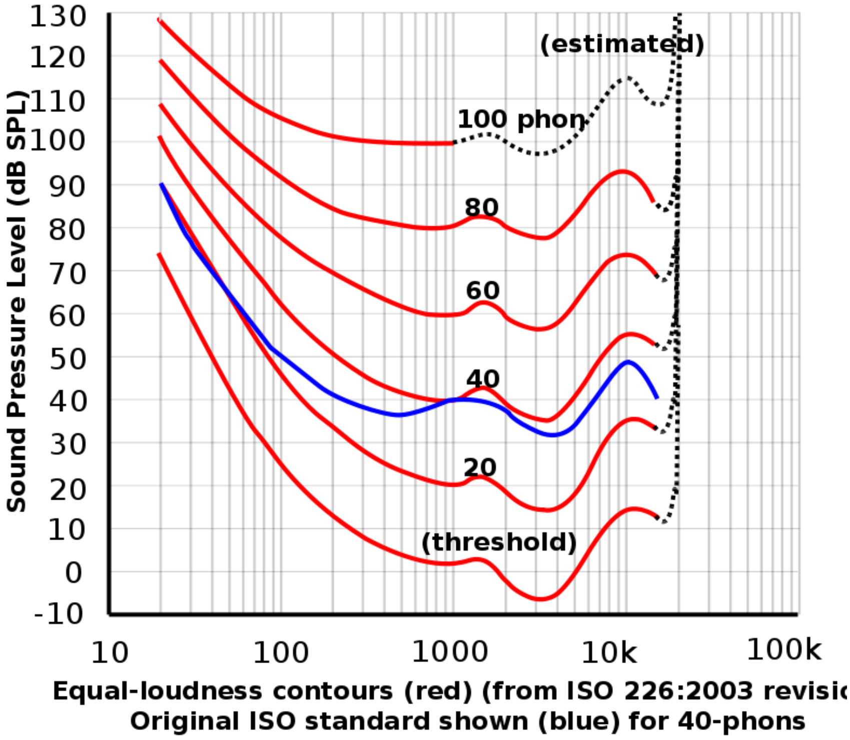
1. Introduction

Humans are exposed to sound and vibration from many different sources, including household appliances, transportation systems, machinery, hand-held power tools, and industrial activities such as piling and blasting. Excessive exposure to noisy sound and vibration may lead to discomfort or health related problems, such as hearing loss or spinal disorders. It is therefore important to assess these risks through proper sound and vibration measurements in accordance with the relevant international or national standards.

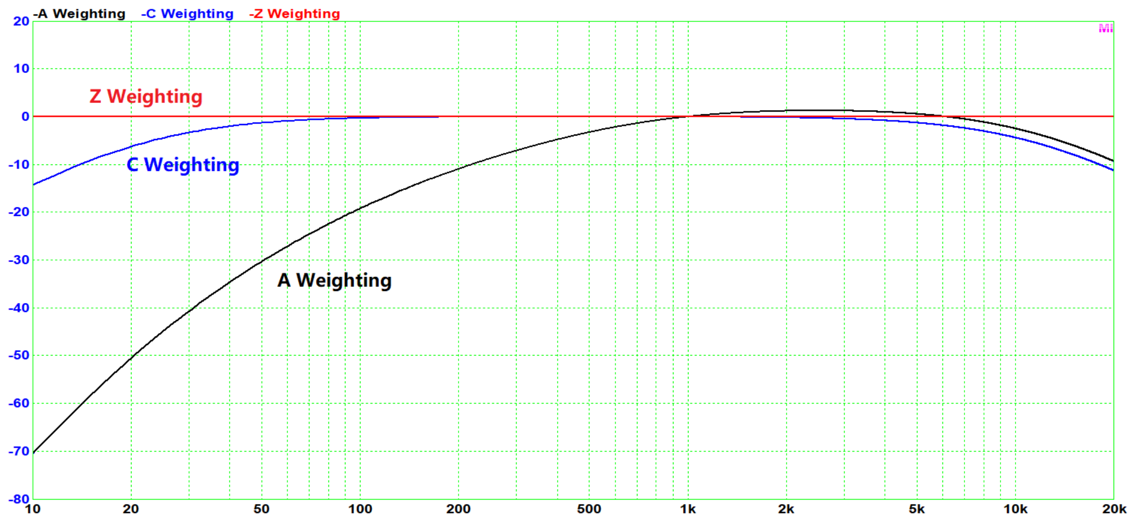
The perception of sound and vibration by humans does not follow a linear pattern with frequency. The human ear is more responsive to sound frequencies between 500Hz and 8kHz, and less sensitive to lower and higher frequencies. Frequency weighting is thus used to correlate the measured sound to the response of the human ear. Three weightings are commonly used: A, C and Z, as defined in IEC 61672. Similarly, the human body is more sensitive to some vibration frequencies than others. Therefore the measured data should be "weighted" to give greater prominence to frequencies where humans are most sensitive. Unlike the case of sound, the sensitivity to vibration is also direction and body posture dependent. Various frequency weighting functions are thus employed to take these factors into account, as stipulated in ISO 2631 and ISO 5349.

2. Frequency Weightings for Sound

Sound loudness is the subjective perception of sound pressure by the human ear. The sensitivity of the human ear does not only depend on frequency but also on sound level, as shown in the equal-loudness graph below. The equal-loudness contour generally becomes flatter as the sound level increases.



A-weighting approximately follows the inverted shape of the equal-loudness contour passing through 40 dB at 1 kHz (i.e. a sound loudness level of 40 phon), and is commonly used for measuring environmental noise and assessing its impact on human perception, while C-weighting is based on the equal-loudness contour passing through 100 dB at 1kHz (i.e. a sound loudness level of 100 phon), and is often used in measuring high-level industrial noise. Unlike A and C weightings, Z-weighing has a flat frequency response between 10Hz and 20kHz. It is used for unweighted measurements. There are also other frequency weighting schemes available, such as B-, D-, G-, and ITU-R 468-weightings. B-weighting is based on the equal-loudness contour of 70 phon. D-weighting was specially designed for non-bypass-type jet engines found only in military aircraft. G weighting is used for measurement in the infrasound range from 8 Hz to about 40Hz. ITU-R 468 noise weighting was developed to more accurately reflect the subjective loudness of all types of noise, as opposed to pure tones. It is widely used in Europe, especially in broadcasting industry. The following graph shows the most commonly used A, C and Z weighting curves.



A, C and Z weighting curves

Both IEC 61672 and ANSI S1.43 provide analytical expressions for C and A frequency weightings in dB as follows. The C-weighting characteristic is realized by two low-frequency poles at frequency f_1, two high-frequency poles at frequency f_4, and two zeros at 0 Hz. With these poles and zeros, the power response for the C-weighting characteristic, relative to the response at the reference frequency f_r of 1 kHz, will be reduced by ½ (approximately -3 dB) at $f_L = 10^{1.5}$ Hz and $f_H = 10^{3.9}$ Hz. The A-weighting characteristic is realized by adding two coupled first-order high-pass filters to the C-weighting characteristic. This is equivalent to the addition of two zeros at 0 Hz and poles at frequencies f_2 and f_3. For each high-pass filter, the cut-off frequency is given by $f_A = 10^{2.45}$ Hz.

$$C(f) = 10\lg\left[\frac{f_4^2 f^2}{\left(f^2+f_1^2\right)\left(f^2+f_4^2\right)}\right]^2 \text{dB} - C_{1000}$$

$$A(f) = 10\lg\left[\frac{f_4^2 f^4}{\left(f^2+f_1^2\right)\left(f^2+f_2^2\right)^{1/2}\left(f^2+f_3^2\right)^{1/2}\left(f^2+f_4^2\right)}\right]^2 \text{dB} - A_{1000}$$

where:

$$f_1 = \left(\frac{-b-\sqrt{b^2-4c}}{2}\right)^{1/2} \qquad f_2 = \left(\frac{3-\sqrt{5}}{2}\right)f_A$$

$$f_4 = \left(\frac{-b+\sqrt{b^2-4c}}{2}\right)^{1/2} \qquad f_3 = \left(\frac{3+\sqrt{5}}{2}\right)f_A$$

$$b = \frac{1}{1-\sqrt{0.5}}\left[f_r^2 + \frac{f_L^2 f_H^2}{f_r^2} - \sqrt{0.5}\left(f_L^2+f_H^2\right)\right] \qquad c = f_L^2 f_H^2$$

f_r = 1000 Hz, $f_A = 10^{2.45}$ Hz, $f_L = 10^{1.5}$ Hz, $f_H = 10^{3.9}$ Hz, A_{1000} and C_{1000} are normalization constants which provide a frequency weighting of 0dB at 1000Hz for A weighting and C weighting respectively. It can be calculated from the above equations that f_1 = 20.60 Hz, f_2 = 107.7 Hz, f_3 = 737.9 Hz, f_4 = 12194 Hz, C_{1000} = -0.062 dB, A_{1000} = -2.000 dB.

ANSI S1.42 provides s-domain transfer functions to realize the above magnitude frequency responses for C and A weightings using the analog method.

$$H_C(s) = G_C \frac{\omega_4^2 \cdot s^2}{\left(s+\omega_1\right)^2 \cdot \left(s+\omega_4\right)^2}$$

$$H_A(s) = G_A \frac{\omega_4^2 \cdot s^4}{\left(s+\omega_1\right)^2 \cdot \left(s+\omega_2\right) \cdot \left(s+\omega_3\right) \cdot \left(s+\omega_4\right)^2}$$

where $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$, $\omega_3 = 2\pi f_3$, $\omega_4 = 2\pi f_4$, $G_c = 10^{0.062/20}$ and $G_A = 10^{2/20}$.

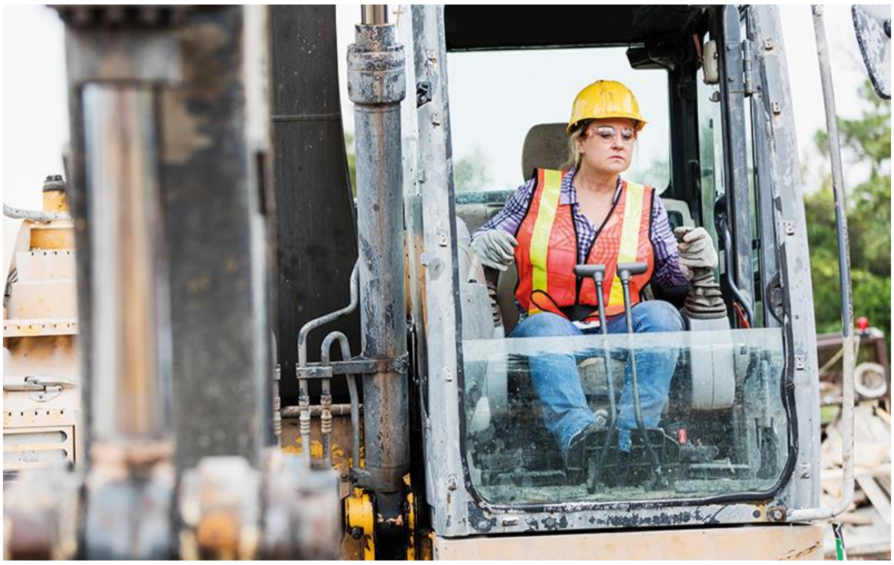
The following table shows the calculated A, C and Z frequency weightings for the centerline frequencies of 1/3 octave bands from 10Hz to 20kHz, where $f = fr \times 10^{(n-30)/10)}$, $n = 10, 11, \ldots, 43$.

Nominal frequency Hz	**Frequency weightings** dB		
	A	**C**	**Z**
10	-70,4	-14,3	0,0
12,5	-63,4	-11,2	0,0
16	-56,7	-8,5	0,0
20	-50,5	-6,2	0,0
25	-44,7	-4,4	0,0
31,5	-39,4	-3,0	0,0
40	-34,6	-2,0	0,0
50	-30,2	-1,3	0,0
63	-26,2	-0,8	0,0
80	-22,5	-0,5	0,0
100	-19,1	-0,3	0,0
125	-16,1	-0,2	0,0
160	-13,4	-0,1	0,0
200	-10,9	0,0	0,0
250	-8,6	0,0	0,0
315	-6,6	0,0	0,0
400	-4,8	0,0	0,0
500	-3,2	0,0	0,0
630	-1,9	0,0	0,0
800	-0,8	0,0	0,0
1 000	0	0	0
1 250	+0,6	0,0	0,0
1 600	+1,0	-0,1	0,0
2 000	+1,2	-0,2	0,0
2 500	+1,3	-0,3	0,0
3 150	+1,2	-0,5	0,0
4 000	+1,0	-0,8	0,0
5 000	+0,5	-1,3	0,0
6 300	-0,1	-2,0	0,0
8 000	-1,1	-3,0	0,0
10 000	-2,5	-4,4	0,0
12 500	-4,3	-6,2	0,0
16 000	-6,6	-8,5	0,0
20 000	-9,3	-11,2	0,0

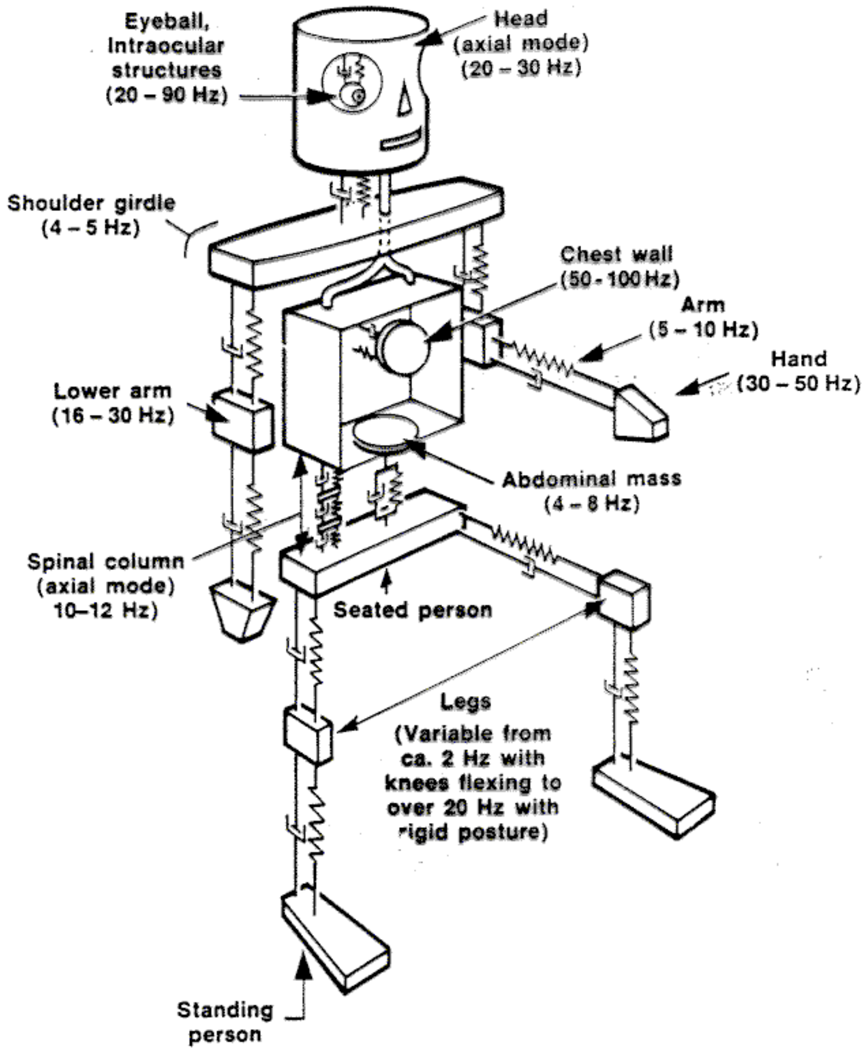
A, C and Z weightings in 1/3 octave bands

It should be noted that each frequency weighting scheme mentioned above is only a function of frequency and does not take into account the variation of hearing sensitivity with sound level and auditory masking effects. These frequency weighting functions are relatively straightforward to implement, well defined in standards, and thus have been widely used. However, it has long been known that these conventional acoustic metrics don’t correlate well with perceptions of sound quality by humans. The term sound quality here refers to the overall experience of the information in the sound that leads to a person’s liking it or not, or that leads to a perception of the non-acoustical qualities of the device emitting the sound (that is, engine power, robust construction, etc.). For some demanding applications, more advanced sound quality metrics such as loudness, loudness level, sharpness, and articulation index are required. These metrics are based on more complex psychoacoustic models and will be discussed elsewhere.

3. Frequency Weightings for Vibration



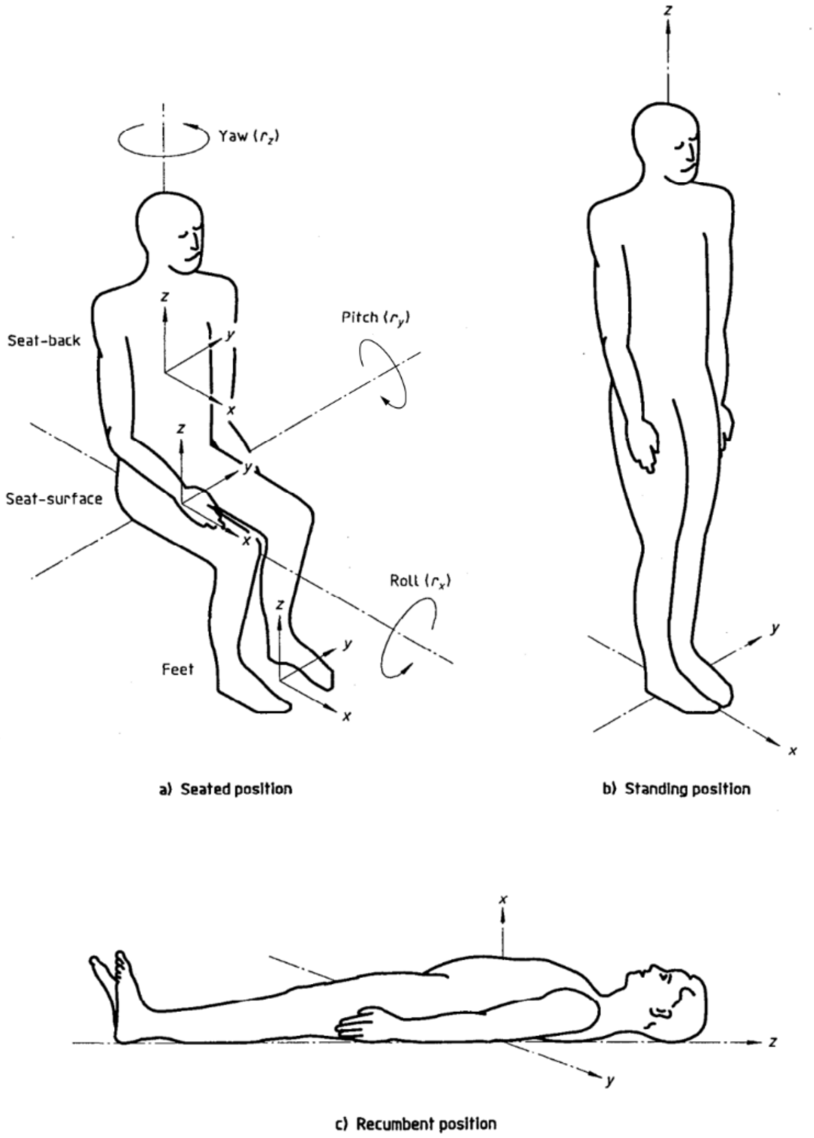
Human vibration refers to the effect of mechanical vibration on the human body. There are two main types of human vibration: whole-body vibration and hand-arm vibration. Whole-body vibration is transmitted to the human body as a whole, generally through the supporting surfaces: the feet of a standing person, the buttocks, back and feet of a seated person, or the supporting area of a recumbent person. This type of vibration is found in vehicles, machinery, buildings and the vicinity of working machinery. The most prominent examples are off-road vehicles, jet boats, helicopters, and jets. Hand-arm vibration is transmitted to the hands and arms. It is mainly experienced by operators of hand-held power tools. The whole-body system and the hand-arm system are "mechanically different", therefore they are studied separately. The human body can be viewed as a set of spring-mass-damper systems, each with a unique natural frequency. The following figure illustrates a simplified mechanical model of the human body.



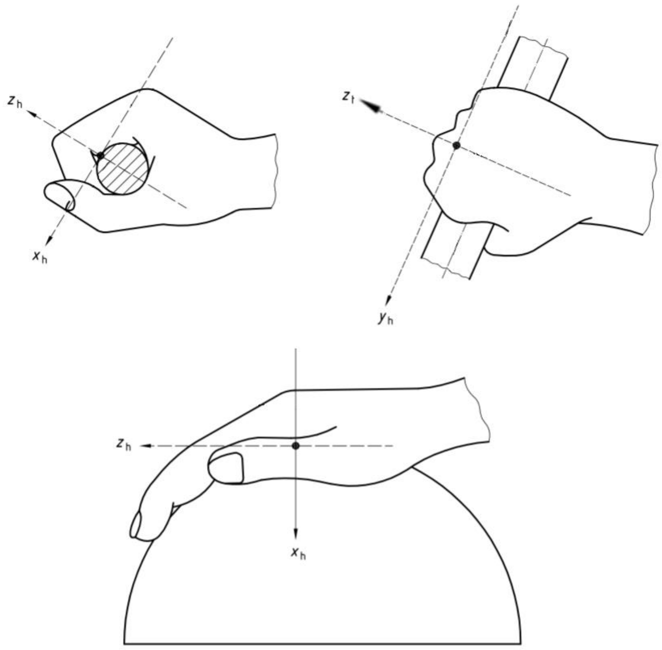
Mechanical Model of Human Body Showing Resonance Frequency Ranges of Various Body Sections

The model shows that different parts of the human body have different resonance frequencies. Certain vibration frequencies can be more damaging to some body parts than others. For example, vibrations between 20Hz and 30Hz can cause resonance between the head and shoulders with vibration amplification of up to 350%! The human body is a strongly damped system and therefore, when a part of it is excited at its natural frequency, it will resonate over a range of frequencies instead of at a single frequency.

The sensitivity of the human body to mechanical vibration depends on frequency, exposure location on the human body, direction of translational or rectilinear vibration, axis of rotational vibration, and posture of the human body. Various frequency weighting functions have been developed to take these factors into account. The following figures show the basicentric coordinate systems of the human body and hand.



<u>Basicentric axes of human body</u>



Basicentric axes of hand

A summary table showing the weighting functions recommended for various locations, postures, and directions for vibration evaluation with respect to health, comfort, perception, and motion sickness is presented below.

Frequency Weighting	Health	Comfort	Perception	Motion Sickness	Nominal Frequency Range Hz	Standard
W_k	*Z* axis, seat surface	*Z* axis, seat surface *Z* axis, standing Vertical recumbent (except head) *X*, *Y*, *Z* axes, feet (sitting)	*Z* axis, seat surface *Z* axis, standing Vertical recumbent (except head)	___	0.5~80	ISO 2631-1
W_d	*X* axis, seat surface *Y* axis, seat surface	*X* axis, seat surface *Y* axis, seat surface *X*, *Y* axes, standing Horizontal recumbent *Y*, *Z* axes, seat back	*X* axis, seat surface *Y* axis, seat surface *X*, *Y* axes, standing Horizontal recumbent	___	0.5~80	ISO 2631-1
W_f	___	___	___	Vertical	0.1~0.5	ISO 2631-1
W_c	*X* axis, seat back	*X* axis, seat back	*X* axis, seat back	___	0.5~80	ISO 2631-1
W_e	___	r_x, r_y, r_z axes, seat surface	r_x, r_y, r_z axes, seat surface	___	0.5~80	ISO 2631-1
W_j	___	Vertical recumbent (head)	Vertical recumbent (head)	___	0.5~80	ISO 2631-1
W_m	___	Whole-body vibration on buildings, all directions	non-specific posture in	___	1~80	ISO 2631-2
W_b	___	Whole-body vibration in fixed-guideway transport systems, *Z* axis	___	___	0.5~80	ISO 2631-4
W_h	Hand-arm vibration, all directions		___	___	8~1000 Hz	ISO 5349-1

There are three principal frequency weightings for whole-body vibration: W_k, W_d and W_f. W_k is for *Z* direction and for vertical recumbent direction (except head), while W_d is for *X* and *Y* directions and for horizontal recumbent direction. Both of them are used for assessing whole-body vibrations concerning health, comfort and perception. However, for evaluations related to motion sickness, W_f should be adopted instead. It focuses on low-frequency whole-body vibration.

W_c, W_e and W_j are additional frequency weightings for special cases of whole-body vibration: seat-back vibration, rotational vibration, and vibration under the head of a recumbent person, respectively.

W_m is the frequency weighting for whole-body vibration and shock in buildings with respect to the comfort and annoyance of the occupants, where the posture of an occupant does not need to be defined.

W_b is the frequency weighting for the evaluation of whole-body vibration on the comfort of passengers and crew in fixed-guideway transport systems.

W_h is the frequency weighting for the measurement and evaluation of hand transmitted vibration from vibrating tools, machinery or workpieces, on the comfort, proficiency and health of the operator.

Generally, all the frequency weightings mentioned above can be achieved by cascading four analog filters: a high-pass filter, a low-pass filter, an acceleration-velocity transition filter, and an upwards step filter, as defined in the respective ISO standards as well as ISO 8041.

Band-limiting is achieved through the combination of a high-pass filter and a low-pass filter. Both of them are second-order Butterworth filters. Their transfer functions are as follows.

High-Pass Filter:

$$H_{\mathrm{h}}(s) = \frac{1}{1 + \frac{\omega_1}{Q_1 s} + \left(\frac{\omega_1}{s}\right)^2}$$

Low-Pass Filter:

$$H_{\mathrm{l}}(s) = \frac{1}{1 + \frac{s}{Q_2 \omega_2} + \left(\frac{s}{\omega_2}\right)^2}$$

The acceleration-velocity transition filter is proportional to acceleration at lower frequencies and to velocity at higher frequencies. Its transfer function is as follows.

Acceleration-Velocity Transition Filter:

$$H_{\mathrm{t}}(s) = \frac{\left(1 + \frac{s}{\omega_3}\right) K}{1 + \frac{s}{Q_4 \omega_4} + \left(\frac{s}{\omega_4}\right)^2}$$

Note: $H_t(s)$ = K when f_3 and f_4 (ω_3 and ω_4) equal to infinity.

The upward-step filter has a steepness of approximately 6 dB per octave and is proportional to jerk. Its transfer function is as follows.

Upward-Step Filter:

$$H_s(s)=\frac{1+\frac{s}{Q_5\omega_5}+\left(\frac{s}{\omega_5}\right)^2}{1+\frac{s}{Q_6\omega_6}+\left(\frac{s}{\omega_6}\right)^2}\left(\frac{\omega_5}{\omega_6}\right)^2$$

Note: $H_s(s)$ = 1 when f_5 and f_6 (ω_5 and ω_6) equal to infinity.

The overall frequency weighting transfer function is the product of all the above four transfer functions, that is:

$$H(s) = H_h(s)\cdot H_l(s)\cdot H_t(s)\cdot H_s(s)$$

where s is the variable of the Laplace transform. The product $H_h(s)\cdot H_l(s)$ represents the band-limiting transfer function. It is the same for all weightings except W_f, W_h and W_m. The product $H_t(s)\cdot H_s(s)$ represents the actual weighting transfer function for a certain application. $H_t(s)$ = 1 for weighting W_j and $H_s(s)$ = 1 for weightings W_c, W_d, W_e, W_m, *and* W_h. Let $s = jw = j2\pi f$, we obtain the overall frequency response:

$$H(jw) = H_h(jw)\cdot H_l(jw)\cdot H_t(jw)\cdot H_s(jw)$$

Then the magnitude frequency response would be:

$$|H(jw)| = |H_h(jw)|\cdot|H_l(jw)|\cdot|H_t(jw)|\cdot|H_s(jw)|$$
or
$$|H(f)| = |H_h(f)|\cdot|H_l(f)|\cdot|H_t(f)|\cdot|H_s(f)|$$

It can be derived that:

$$|H_h(f)| = \sqrt{\frac{f^4}{f^4+f_1^4}}$$

$$|H_l(f)| = \sqrt{\frac{f_2^4}{f^4+f_2^4}}$$

$$|H_t(f)| = \sqrt{\frac{f^2+f_3^2}{f_3^2}}\times\sqrt{\frac{f_4^4Q_4^2}{f^4Q_4^2+f^2f_4^2(1-2Q_4^2)+f_4^4Q_4^2}}\times K$$

$$|H_s(f)| = \frac{Q_6}{Q_5} \times \sqrt{\frac{f^4Q_5^2 + f^2f_5^2(1-2Q_5^2) + f_5^4Q_5^2}{f^4Q_6^2 + f^2f_6^2(1-2Q_6^2) + f_6^4Q_6^2}}$$

The parameters of the transfer functions above are listed in the table below.

Weighting	Band-limiting				A-V Transition			Upward Step				Gain
	f_1 Hz	Q_1	f_2 Hz	Q_2	f_3 Hz	f_4 Hz	Q_4	f_5 Hz	Q_5	f_6 Hz	Q_6	K
W_k	0.4	$2^{-1/2}$	100	$2^{-1/2}$	12.5	12.5	0.63	2.37	0.91	3.35	0.91	1
W_d	0.4	$2^{-1/2}$	100	$2^{-1/2}$	2	2	0.63	∞	1	∞	1	1
W_f	0.08	$2^{-1/2}$	0.63	$2^{-1/2}$	∞	0.25	0.86	0.0625	0.80	0.10	0.80	1
W_c	0.4	$2^{-1/2}$	100	$2^{-1/2}$	8	8	0.63	∞	1	∞	1	1
W_e	0.4	$2^{-1/2}$	100	$2^{-1/2}$	1	1	0.63	∞	1	∞	1	1
W_j	0.4	$2^{-1/2}$	100	$2^{-1/2}$	∞	∞	1	3.75	0.91	5.32	0.91	1
W_m	$10^{-0.1}$	$2^{-1/2}$	100	$2^{-1/2}$	1/ (0.028×2π)	1/ (0.028×2π)	0.5	∞	1	∞	1	1
W_b	0.4	$2^{-1/2}$	100	$2^{-1/2}$	16	16	0.55	2.5	0.9	4	0.95	1.024
W_h	$10^{8/10}$	$2^{-1/2}$	$10^{31/10}$	$2^{-1/2}$	100/(2π)	100/(2π)	0.64	∞	1	∞	1	1

Parameters of the Transfer Functions of Vibration Frequency Weightings

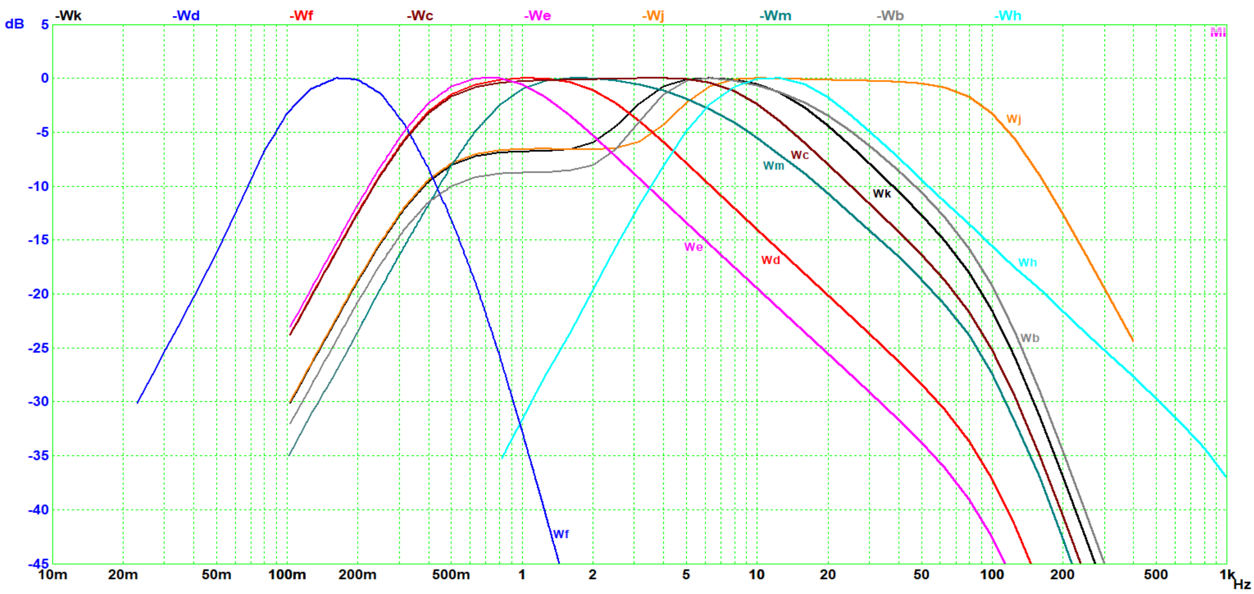
The following table shows the calculated frequency weightings for the centerline frequencies of 1/3 octave bands from 0.02Hz to 4kHz.

Nominal Frequency f Hz	W_k dB	W_d dB	W_f dB	W_c dB	W_e dB	W_j dB	W_m dB	W_b dB	W_h dB
0.02			-32.33						
0.025			-28.48						
0.0315			-24.47						
0.04			-20.25						
0.05			-16.1						
0.063			-11.49						
0.08			-6.73						
0.1	-30.11	-24.09	-3.16	-24.11	-24.08	-30.18	-36	-32.04	
0.125	-26.26	-20.24	-0.96	-20.25	-20.22	-26.32	-32	-28.2	
0.16	-22.05	-16.01	0.05	-16.03	-15.98	-22.11	-28.01	-23.98	
0.2	-18.33	-12.28	-0.07	-12.3	-12.23	-18.38	-24.02	-20.23	
0.25	-14.81	-8.75	-1.37	-8.78	-8.67	-14.86	-20.05	-16.71	
0.315	-11.6	-5.52	-4.17	-5.56	-5.41	-11.65	-16.12	-13.51	
0.4	-9.07	-2.94	-8.31	-3.01	-2.81	-9.1	-12.29	-10.98	
0.5	-7.57	-1.38	-13	-1.48	-1.29	-7.6	-8.67	-9.53	
0.63	-6.77	-0.5	-18.69	-0.64	-0.55	-6.78	-5.51	-8.71	
0.8	-6.43	-0.07	-25.51	-0.24	-0.53	-6.42	-3.09	-8.38	-36
1	-6.33	0.1	-32.57	-0.08	-1.11	-6.3	-1.59	-8.29	-31.99
1.25	-6.29	0.07	-40.02	0	-2.25	-6.28	-0.85	-8.27	-27.99
1.6	-6.12	-0.28	-48.47	0.06	-3.99	-6.32	-0.59	-8.07	-23.99
2	-5.49	-1.01	-56.19	0.1	-5.82	-6.34	-0.61	-7.6	-20.01
2.5	-4.01	-2.2	-63.93	0.15	-7.77	-6.22	-0.82	-6.13	-16.05
3.15	-1.9	-3.85	-71.96	0.19	-9.81	-5.62	-1.19	-3.58	-12.18
4	-0.29	-5.82	-80.26	0.2	-11.93	-4.04	-1.74	-1.02	-8.51
5	0.33	-7.76		0.11	-13.91	-2.01	-2.5	0.21	-5.27
6.3	0.46	-9.81		-0.23	-15.94	-0.48	-3.49	0.46	-2.77
8	0.31	-11.93		-1	-18.03	0.15	-4.7	0.21	-1.18

10	-0.1	-13.91		-2.2	-19.98	0.26	-6.12	-0.23	-0.43
12.5	-0.89	-15.87		-3.79	-21.93	0.22	-7.71	-0.85	-0.38
16	-2.28	-18.03		-5.82	-24.08	0.16	-9.44	-1.83	-0.96
20	-3.93	-19.99		-7.77	-26.02	0.1	-11.25	-3	-2.14
25	-5.8	-21.94		-9.76	-27.97	0.06	-13.14	-4.44	-3.78
31.5	-7.86	-23.98		-11.84	-30.01	0	-15.09	-6.16	-5.69
40	-10.05	-26.13		-14.02	-32.15	-0.08	-17.1	-8.11	-7.72
50	-12.19	-28.22		-16.13	-34.24	-0.24	-19.23	-10.09	-9.78
63	-14.61	-30.6		-18.53	-36.62	-0.62	-21.58	-12.43	-11.83
80	-17.56	-33.53		-21.47	-39.55	-1.48	-24.38	-15.34	-13.88
100	-21.04	-36.99		-24.94	-43.01	-3.01	-27.93	-18.72	-15.91
125	-25.35	-41.28		-29.24	-47.31	-5.36	-32.37	-23	-17.93
160	-30.91	-46.84		-34.8	-52.86	-8.78	-37.55	-28.56	-19.94
200	-36.38	-52.3		-40.26	-58.33	-12.3	-43.18	-34.03	-21.95
250	-42.04	-57.97		-45.92	-63.99	-16.03	-49.02	-39.69	-23.96
315	-48	-63.92		-51.88	-69.94	-19.98	-54.95	-45.65	-25.97
400	-54.2	-70.12		-58.08	-76.14	-24.1	-60.92	-51.84	-28
500									-30.07
630									-32.23
800									-34.6
1000									-37.42
1250									-40.97
1600									-45.42
2000									-50.60
2500									-56.23
3150									-62.07
4000									-68.01

W_k, W_d, W_f, W_c, W_e, W_j, W_m, W_b, and W_h weightings in 1/3 octave bands

The following graph shows the respective weighting curves.



W_k, W_d, W_f, W_c, W_e, W_j, W_m, W_b, and W_h weighting curves

4. Frequency Weighted Values for Evaluation of Sound and Vibration

4.1 Frequency-weighted Root-Mean-Square (RMS) Value

Frequency-weighted root-mean-square (RMS) value is defined by the following linear time-averaging expression:

$$A_w = \left(\frac{1}{T}\int_0^T a_w^2(t)dt\right)^{\frac{1}{2}}$$

where $a_w(t)$ represents the translational or rotational frequency-weighted acceleration in a specified direction or around a specified axis for vibration measurements, or sound pressure for sound measurements, as a function of time t, and T denotes the duration of the measurement.

This value is utilized as the basic evaluation criterion for vibration. However, in cases where vibration exhibits a frequency-weighted crest factor exceeding 9, it may underestimate the impact of vibration on human beings. One of the alternative measures described below should be determined as an additional evaluation criterion – the running RMS value or the fourth power Vibration Dose Value (VDV).

4.2 Frequency-weighted Running Root-Mean-Square (RMS) Value

Frequency-weighted running root-mean-square (RMS) value is defined by the following linear time-averaging expression:

$$A_{wT}(t) = \left(\frac{1}{T}\int_{t-T}^{t} a_w^2(\xi)d\xi\right)^{\frac{1}{2}}$$

where T is the integration time for running averaging, ξ the dummy integration time variable, and t the time of observation (instantaneous time). The linear time averaging above can be approximated by exponential time averaging (i.e. time-weighting) as follows.

$$A_{w\tau}(t) = \left(\frac{1}{\tau}\int_{-\infty}^{t} a_w^2(\xi)\exp\left(\frac{\xi - t}{\tau}\right)d\xi\right)^{\frac{1}{2}}$$

where τ is the time constant. The difference in result is very small for application to shocks of a short duration compared to τ, and somewhat larger (up to 30%) when applied to shocks and transients of longer duration.

4.3 Frequency-weighted Level

Frequency-weighted level is defined as:

$$L_w = 20lg\left(\frac{A_w}{A_0}\right)\ dB$$

where A_0 is the reference acceleration (10^{-6} m/s^2 for translational acceleration as defined in ISO 1683) for vibration measurements, or reference sound pressure (20 μPa in air, 1 μPa in water) for sound measurements.

In sound level measurements, when A_w represents a linearly time-averaged value, L_w is termed the equivalent continuous sound level, often denoted with "T" or "eq" added behind the weighting symbol in the subscript. For example, both L_{AT} and L_{Aeq} represent an A-weighted equivalent continuous sound level. A_w can also be an exponentially time-averaged (or time-weighted) value associated with a time constant, i.e. $A_{w\tau}$. There are typically three standard time weightings: I (Impulse), F (Fast), and S (Slow), with time constants of 35ms, 125ms, and 1s, respectively. In such cases, a time weighting symbol should be added to L_w. For example, L_{AF} denotes an A-weighted "Fast" time-weighted sound level.

4.4 Maximum Transient Value

Maximum Transient Vibration Value (MTVV) is defined as the maximum value of the running RMS vibration acceleration value, i.e. $MTVV = max[A_{wT}(t)]$ or $MTVV = max[A_{w\tau}(t)]$ when the integration time T or τ is equal to 1 s (corresponding to the integration time constant, Slow, in sound level measurements). When $MTVV/A_w$>1.5, $MTVV$ should also be reported as an additional evaluation criterion.

As a comparison, in sound measurements, maximum time-averaged or time-weighted sound level is often measured, such as L_{AFmax}, L_{ASmax}, L_{CFmax}, and L_{CSmax}.

4.5 Motion Sickness Dose Value (MSDV) and Sound Exposure

Both Motion Sickness Dose Value (MSDV) and Sound Exposure can be defined by the following expression:

$$\text{MSDV or Sound Exposure}(E_{wT}) = \left(\int_0^T a_w^2(t)dt\right)^{\frac{1}{2}}$$

where T is the total period for which the motion or sound exposure occurs.

In sound measurements, sound exposure is often converted to sound exposure level as follows:

$$L_{wET} = 10lg\left(\frac{\int_0^T a_w^2(t)dt}{A_0^2 T_0}\right)\ dB$$

where A_0 is the reference sound pressure (20 μPa in air), and $T_0 = 1$ s.

4.6 Vibration Dose Value (VDV)

Vibration Dose Value is defined as follows:

$$\mathrm{VDV} = \left(\int_0^T a_w^4(t)dt\right)^{\frac{1}{4}}$$

where T is the total period for which the vibration exposure occurs. When $VDV/(A_w T^{1/4})>1.75$, *VDV* should also be reported as an additional evaluation criterion.

VDV can be linearly time-averaged to yield the so-called Frequency Weighted Root-Mean-Quad (RMQ) value as follows.

$$\mathrm{RMQ} = \left(\frac{1}{T}\int_0^T a_w^4(t)dt\right)^{\frac{1}{4}}$$

RMQ is more sensitive to peaks than the RMS value.

4.7 Peak Value

Maximum modulus of the instantaneous (positive and negative) peak value of the frequency weighted acceleration or sound pressure.

In sound measurements, peak sound pressure is often converted to peak sound level using the reference sound pressure. For example, L_{Cpeak} represents the C-weighted peak sound level.

4.8 Vibration Total Value

Vibration Total Value combines vibration from three axes of translational vibration, as defined by the following expression:

$$A_{wv} = \sqrt{k_x^2 A_{wx}^2 + k_y^2 A_{wy}^2 + k_z^2 A_{wz}^2}$$

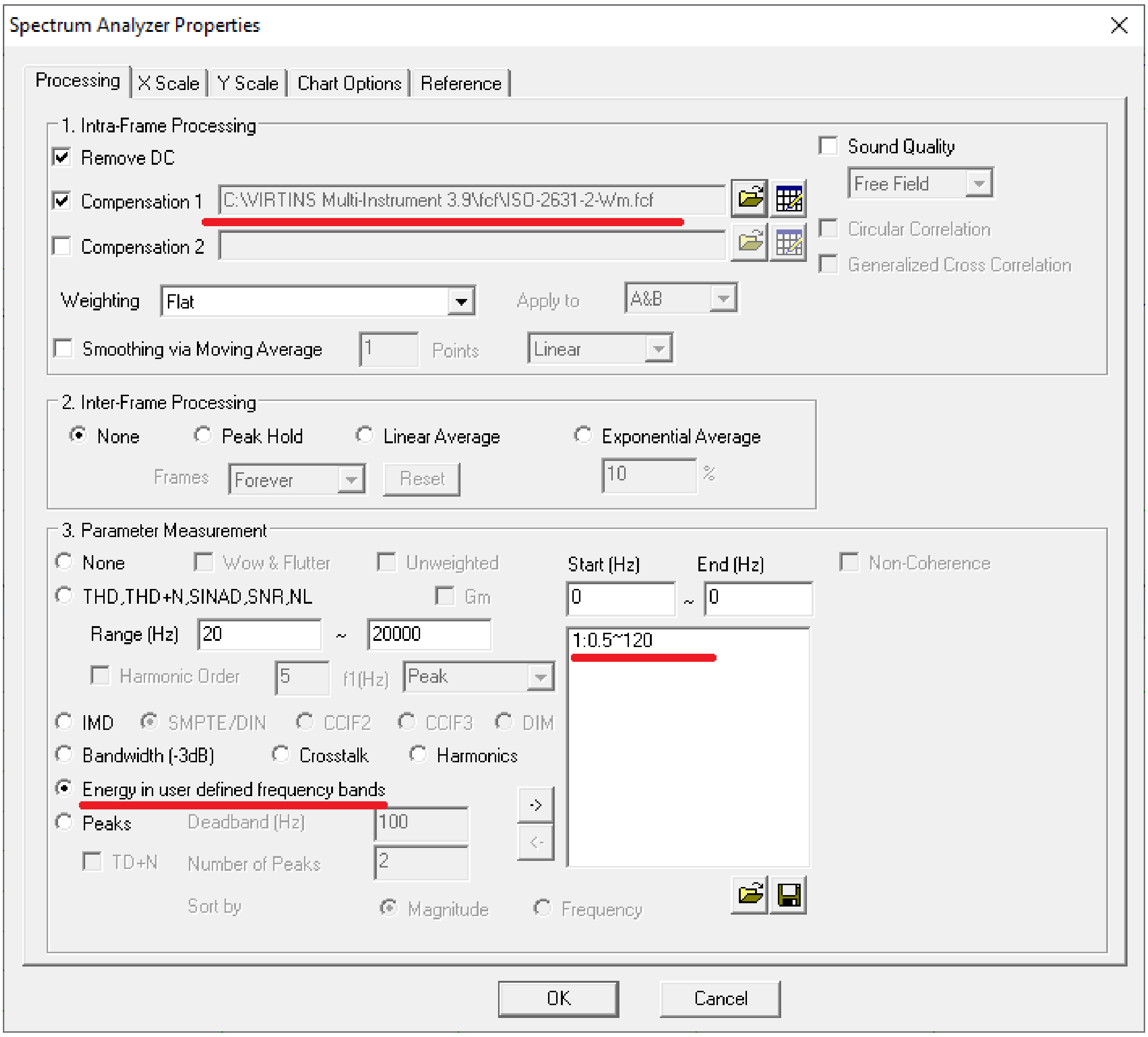
where A_{wx}, A_{wy}, and A_{wz} are the frequency weighted RMS accelerations in the three orthogonal axes *x*, *y* and *z*, respectively. k_x, k_y, and k_z are multiplying constants whose values depend on the measurement application. For example, for hand-arm vibration, $kx = ky = kz = 1$. For assessment of the effect of vibration on the health of seated persons:
x-axis: W_d, $k_x = 1.4$
y-axis: W_d, $k_y = 1.4$
z-axis: W_k, $k_z = 1$

5. Frequency Weighting Implementation

5.1 Frequency Domain Implementation

The frequency-weighted RMS value is the fundamental metric for sound and vibration evaluation. It is defined in the time domain, but it can also be calculated from the frequency domain according to Parseval's theorem. Motion Sickness Dose Value (MSDV) and Sound Exposure can be derived from the RMS value based on the measurement time duration. If only these RMS-related values are of interest, then directly applying frequency weighting on the spectrum data in the frequency domain would be the easier choice.

In Multi-Instrument, this can be done by right clicking the Spectrum Analyzer window, selecting [Spectrum Analyzer Processing]>[Compensation 1], and then loading the respective frequency weighting file. The software comes with CSV text files (called frequency compensation file, *.fcf) containing magnitude frequency response data for all the frequency weightings mentioned previously. The frequency-weighted RMS value can be displayed using the "Energy in User Defined Frequency Bands" function, as shown below.



Frequency Domain Implementation of Frequency Weighting in Multi-Instrument

MSDV and Sound Exposure can be calculated by multiplying the measured frequency-weighted RMS value with the square root of the measurement time duration. This formula can be used to define a User Defined Data Point (UDDP) in Multi-Instrument in order to display the result directly.

The variation of running RMS with time can be captured by configuring the oscilloscope frame width of Multi-Instrument to match the integration time of the running averaging, and keeping the data sampling and analyzing process running. The Maximum Transient Value can be obtained by the Peak Hold function of the DDP Viewer of the software. The above analysis can also be performed after the raw data has been recorded as a WAV file. Unlike the real time analysis, post-analysis uses [File]>[Open Frame by Frame] function and it is possible to specify the frame overlap percentage. The higher the overlap percentage, the closer the running RMS approaches the ideal.

It should be noted that the instantaneous Peak Value, Vibration Dose Value (VDV) as well as Crest Factor (*CF = Peak / RMS*) cannot be obtained through frequency domain implementation of frequency weighting, time domain implementation is required instead.

5.2 Time Domain Implementation

Frequency weighting can be achieved through the use of either analog filters or digital filters in the time domain. Analog filters can be readily designed based on the analog transfer functions in the Laplace domain (s-domain) described previously. However, modern data acquisition systems used for sound and vibration recording are almost all digital, thus frequency weighting through digital filters becomes the logical preference. This eliminates the need for costly and bulky analog filters. There are three types of digital filters: IIR, FIR, and FFT.

When frequency weighting is implemented in the time domain, all the frequency weighted values mentioned previously can be calculated directly from the time domain. In Multi-Instrument, the peak and RMS values are automatically calculated and displayed in the Oscilloscope window. VDV can be derived from the automatically calculated Kurtosis and frequency weighted RMS using the following equation:

$$\mathrm{VDV} = (Kurtosis \times T)^{\frac{1}{4}} \times A_w$$

where *T* is the total period for which the vibration exposure occurs, and *Kurtosis* is defined as:

$$\mathrm{Kurtosis} = \frac{\frac{1}{N}\sum_{i=0}^{N-1} a_w^4[i]}{A_w^4}$$

Note: $a_w(t)$ is assumed to have a mean value of zero, that is, with no DC component. Otherwise, tick the “Remove DC” option in [Oscilloscope Processing] in the software.

The above VDV equation can be used to define a UDDP in Multi-Instrument in order to display VDV directly.

5.2.1 IIR Filter

As the digital counterpart of an analog filter, an Infinite Impulse Response (IIR) filter has an impulse response function which is non-zero over an infinite duration. It is a recursive filter in that the output from the filter is computed by using the current and previous inputs and previous outputs. It has the following form in the time domain:

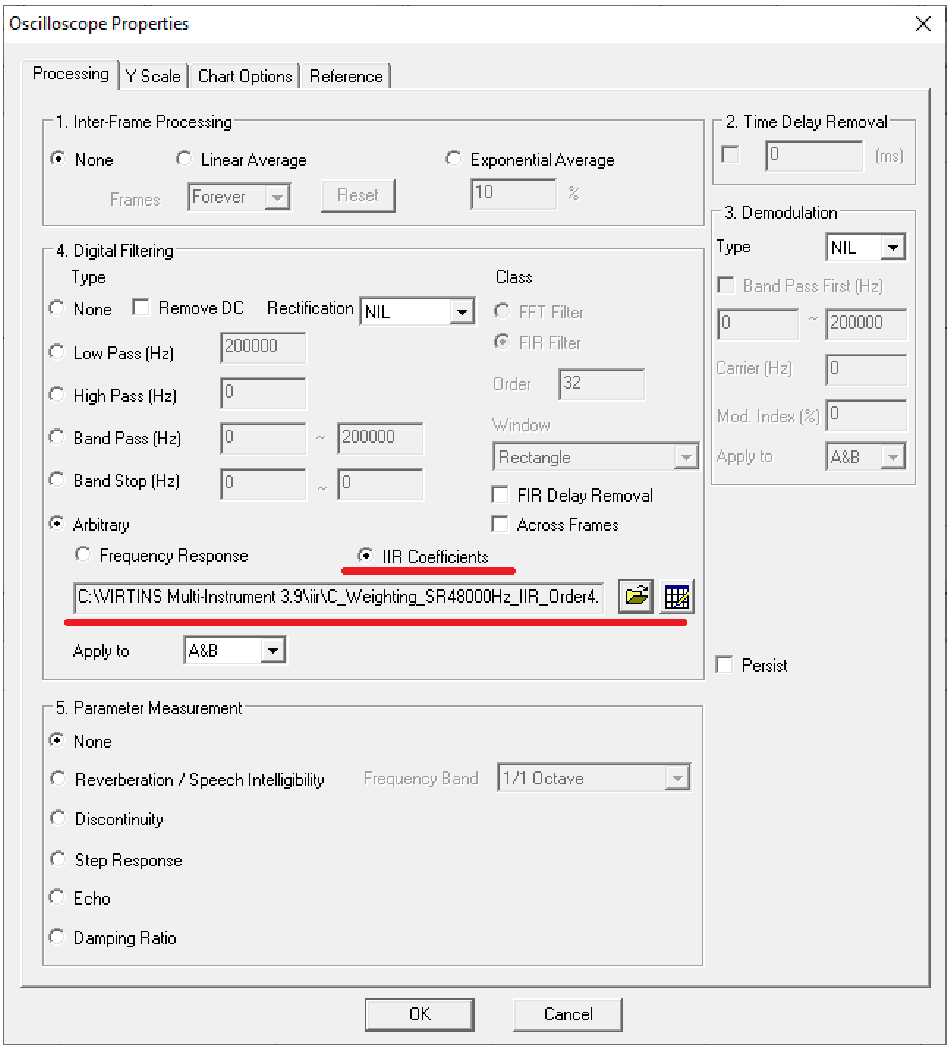
$$\mathrm{y[n]} = \sum_{i=0}^{M} b_i x[n-i] - \sum_{j=1}^{N} a_j y[n-j]$$

where:
x[n] is the input signal,
y[n] is the output signal,
b_i is the so-called feedforward filter coefficients,
a_i is the so-called feedback filter coefficients,
M is the feedforward filter order,
N is the feedback filter order.

Taking z-transform of the above equation yields its transfer function:

$$\mathrm{H[z]} = \frac{\sum_{i=0}^{M} b_i z^{-i}}{1 + \sum_{j=1}^{N} a_j z^{-j}}$$

There are a few methods to derive the equivalent IIR filter from a given analog transfer function in the s-domain, including Bilinear Transform, Impulse Invariance, and Matched Z-Transform. The resulting IIR coefficients for various frequency weighting functions have been reported in many research articles. Multi-Instrument supports IIR filter through the input of IIR coefficients from a CSV text file, as shown below. For post-analysis with [File]>[Open Frame by Frame], “Across Frames” option should be ticked to ensure continuity across frames.



<u>Time Domain Implementation of Frequency Weighting using IIR in Multi-Instrument</u>

An IIR filter requires much less filter coefficients than its equivalent FIR filter. However, it uses feedbacks from the output to the input and thus may lead to instability. It does not have a linear phase response and thus the filtered waveform is distorted, leading to errors in non-RMS parameters such as crest factor, peak value, and VDV. It should be noted that the analog transfer functions specified in the aforementioned standards do not have a linear phase response, either. These standards still lean towards conventional analog filters, which are extremely difficult, if not impossible, to achieve linear phase responses.

5.2.2 FIR Filter

A Finite Impulse Response (FIR) filter has a finite impulse response. It has no feedback and therefore is always stable. It has the following form in the time domain:

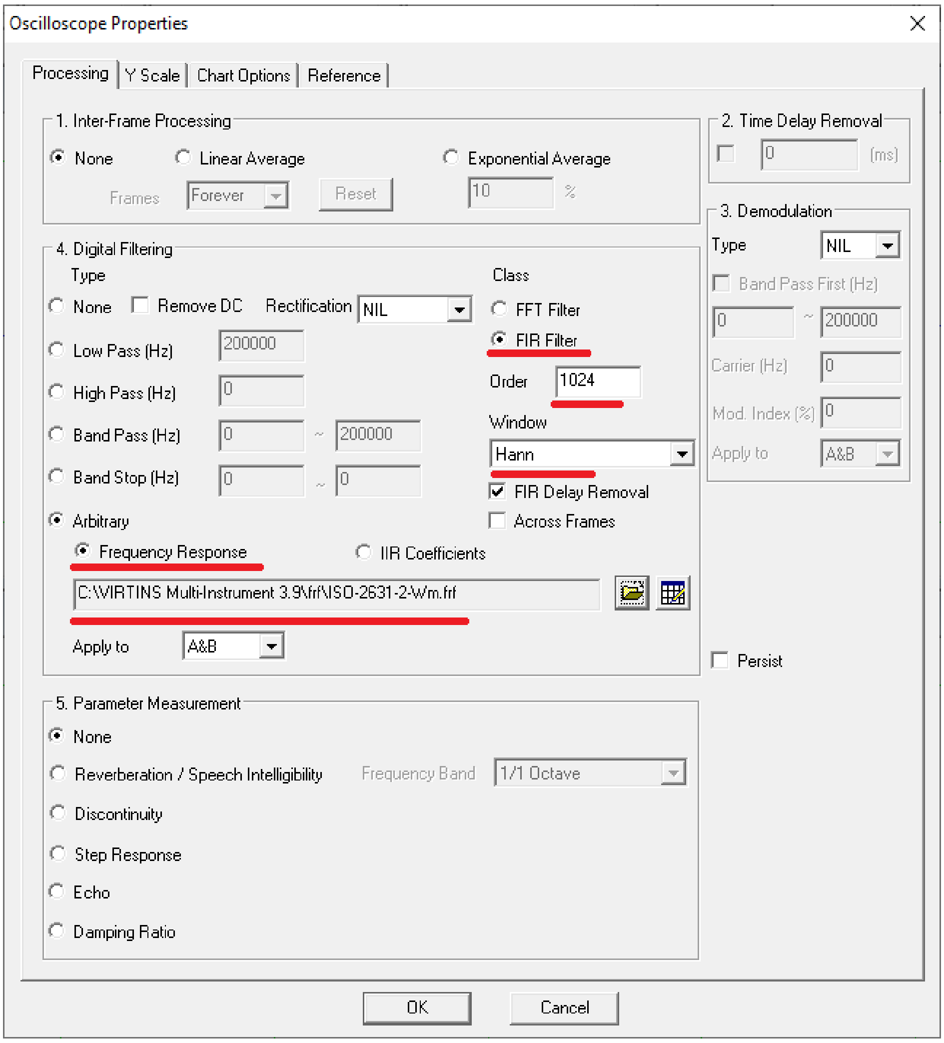
$$\mathrm{y[n]} = \sum_{i=0}^{M} b_i x[n-i]$$

Its transfer function is:

$$\mathrm{H}[z] = \sum_{i=0}^{M} b_i z^{-i}$$

The main disadvantage of a FIR filter is that it would need a large number of coefficients to match the magnitude frequency response of the weighting filter given in the standard. A FIR filter is usually designed to have symmetrical filter coefficients in order to achieve a linear phase response. Given the magnitude frequency response, there are several methods to design a FIR filter, such as the Windowing method.

Multi-Instrument supports the design of five types of FIR filters: low-pass, high-pass, band-pass, band-stop, and arbitrary, based on the specified cut-off frequencies, or the magnitude frequency response provided in a CSV text file. If the FIR filter coefficients are already known, they can be input into the software using the IIR coefficient method described previously. Arbitrary FIR filters should be used for frequency weightings, as shown below. The software comes with CSV text files (called frequency response file, *.frf) containing magnitude frequency response data for all the frequency weightings mentioned previously.



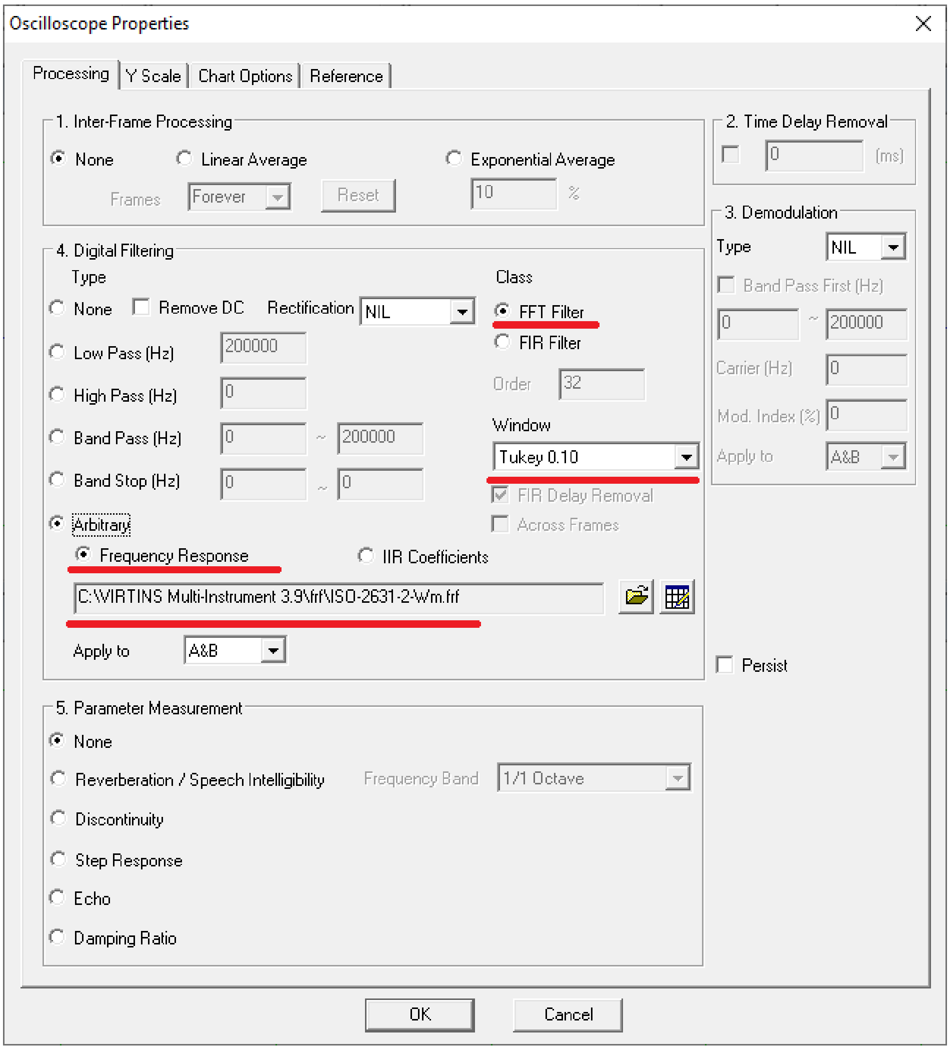
Time Domain Implementation of Frequency Weighting using FIR in Multi-Instrument

Again, for post-analysis with [File]>[Open Frame by Frame], "Across Frames" option should be ticked to ensure the continuity across frames.

5.2.3 FFT Filter

Unlike FIR and IIR filters, the filtering function of a FFT filter is not done in the time domain. Instead, the input signal is transformed from the time domain to the frequency domain using FFT. Its spectrum is then multiplied with the filter's magnitude frequency response and the result is transformed back to the time domain using the inverse FFT. FFT filter has a linear phase response, or more precisely, zero-phase response.

Similar to the case of FIR filters, Multi-Instrument supports five types of FFT filters: low-pass, high-pass, band-pass, band-stop, and arbitrary. Arbitrary FFT filters should be used for frequency weightings, as shown below. Again, the software comes with CSV text files containing magnitude frequency response data for all the frequency weightings mentioned previously.



<u>**Time Domain Implementation of Frequency Weighting using FFT in Multi-Instrument**</u>

Normally Rectangle window function should be used for FFT filters. If there is a need to suppress the boundary effect (e.g. overshoot at both ends of the waveform due to the inherent issues with the periodic extension assumption in FFT), one of the 7 Tukey windows can be applied.

It should be noted that for post-analysis with [File]>[Open Frame by Frame], FFT filters cannot ensure the continuity across frames, unlike IIR and FIR filters.

5.3 Measurement Duration

The duration of measurement shall be sufficient to ensure reasonable statistical precision and to ensure that the sound and vibration is typical of the exposures which are being assessed. For stationary random signals, the measurement accuracy depends on the filter bandwidth and measurement duration. For example, in vibration evaluation, to obtain a measurement error of less than 3 dB at a confidence level of 90% requires a minimum measurement duration of 108s for a lower limiting frequency (LLF) of 1Hz and 227 s for a LLF of 0.5Hz, when the analysis is done with a one-third octave bandwidth.

